

THREE - DIMENSIONAL GEOMETRY

SECTION - A

Questions 1 to 10 carry 1 mark each.

- The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:
 (a) $-1/2$ (b) $1/2$ (c) 2 (d) 3
- The cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$ is:
 (a) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (b) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (c) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (d) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$
- The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are:
 (a) 3, 6, 1 (b) 3, 6, -1 (c) 2, 1, 6 (d) 2, 1, -6
- If a line in space makes angle α, β, γ with the positive direction of coordinate axes then the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is:
 (a) 0 (b) -1 (c) 2 (d) 3
- The direction ratios of three lines l, m, n are given below:
 $2, 3, 4; -7, 2, 2; 4, 7, 7$
 Which of the two pairs of lines are perpendicular?
 (a) l and m ; m and n (b) l and n ; m and n
 (c) l and m ; l and n (d) l and m ; m and n ; l and n
- A line m passes through the point $(-4, 2, -3)$ and is parallel to line n , given by:
 $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$
 The vector equation of line m is given by: $\vec{r} = (-4\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(p\hat{i} + q\hat{j} + r\hat{k})$, where $\lambda \in \mathbb{R}$.
 Which of the following could be the possible values for p, q and r ?
 (a) $p = 4, q = (-2), r = 3$ (b) $p = (-4), q = (-2), r = 3$
 (c) $p = (-2), q = 3, r = (-6)$ (d) $p = 8, q = 4, r = (-3)$
- The lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(6\hat{i} + 9\hat{j} - 18\hat{k})$ (where λ and μ are scalars) are:
 (a) coincident (b) skew (c) intersecting (d) parallel
- If the direction cosines of a line are $\langle \frac{1}{c}, \frac{1}{c}, \frac{1}{c} \rangle$ then:
 (a) $0 < c < 1$ (b) $c > 2$ (c) $c = \pm \sqrt{2}$ (d) $c = \pm \sqrt{3}$

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In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
- (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Reason (R): The sum of squares of the direction cosines of a line is 1.

10. **Assertion (A):** The vector equation of the line passing through the points $(6, -4, 5)$ and $(3, 4, 1)$ is $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + 8\hat{j} + 4\hat{k})$

Reason (R): The vector equation of the line passing through the points \vec{a} and \vec{b} is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Using direction ratios, show that the points $(2, 3, 4)$, $(-1, -2, 1)$ and $(5, 8, 7)$ are collinear.
12. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.
13. Find the value of λ , so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.
14. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z-axis.

OR

Find the angle between the following pair of lines: $\frac{2-x}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and

$\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the equation of a line passing through the point $P(2, -1, 3)$ and perpendicular to the lines:

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

16. Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

OR

A line passes through $(2, -1, 3)$ and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and cartesian form.

17. Find the length and the foot of the perpendicular drawn from the point $(2, -1, 5)$ on the

$$\text{line } \frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

SECTION – D

Questions 18 carry 5 marks.

18. Find the co-ordinates of the foot of the r and the length of the perpendicular drawn from the point

$P(5, 4, 2)$ to the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also, find the image of P in this line.

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SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

A cricket match is organised between the clubs A and B for which a team from each club is chosen. Remaining players of club A and club B are respectively sitting on the lines represented by the

equations $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ and $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$ and to cheer the team of their own clubs.



- Find the vector equation of the line on which players of club A can be seated.
- Write the direction cosines of the line on which players of club B are seated.
- If the line on which players of club A are seated, is perpendicular to the line, whose cartesian equation is $\frac{x-3}{3} = \frac{y-7}{4} = \frac{kz-7}{1}$, then find the value of k.
- Find the angle between the lines on which players of clubs A and B are seated.

20. Case-Study 2: Read the following passage and answer the questions given below.

In the given figure, the vector equation of lines in the extreme left side are $\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(\hat{i} + 3\hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 3\hat{j} + \hat{k})$. There are 8 lines between the two extreme lines and the distance between any two consecutive lines are equal.

If the cartesian equation of finishing line is $7 - x = 3y - 2 = 1 - z$ and position vector of winning player (position shown in figure below) is $2\hat{i} + 3\hat{j} - 3\hat{k}$.



Based on the given information, answer the following questions:

- Find the distance between any two consecutive lines. (2)
- Find the perpendicular distance between the position of winning player and the finishing line. (2)